Mathematics of statistical decision making

and applications to bariatric surgery

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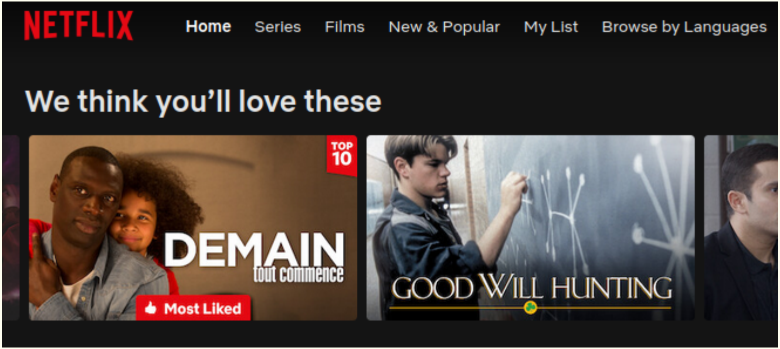
Centre de Recherche en Informatique, Signal et Automatique de Lille

Towards recommender systems in healthcare

Online advertising



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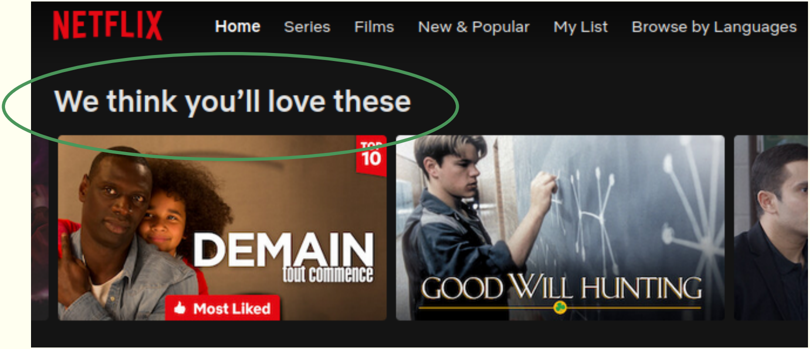

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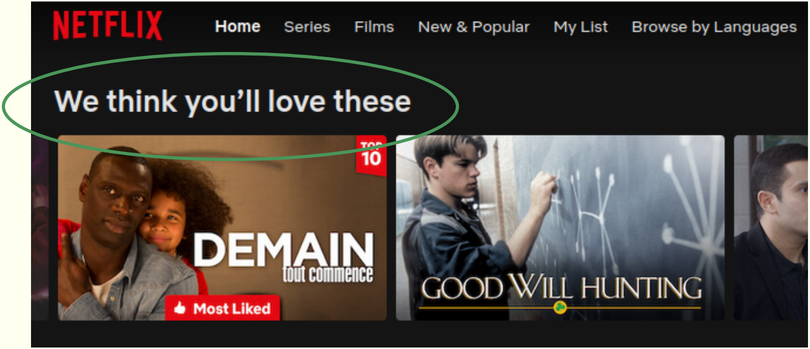


$\hookrightarrow$ from online marketing to healthcare recommendations.

Postoperative follow-up: where to go?

Patient at time t

Patient at time $t-1$

Patient at time $t+1$

Surgery department

General practitioner

Electronic consultation

Postoperative follow-up: when to go?

Patient at time t

Patient at time $t-1$

Patient at time $t+1$

Surgery department

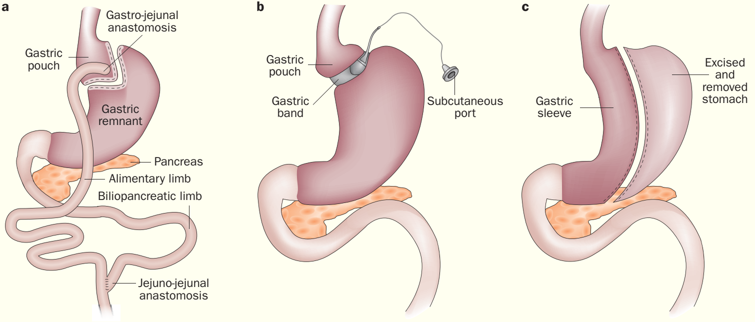
follow-up visit after...

3 months?

6 months?

1 year?

Bariatric surgery



Roux-en-Y gastric bypass

Adjustable gastric band

Sleeve gastrectomy

↪ In France: 60 000 operations each year, 1% of adults.

Miras and Le Roux. Mechanisms underlying weight loss after bariatric surgery. Nature reviews, 2013.

Contributions in medical sciences

Publications in peer-reviewed journals

- **PS**, Pierre Bauvin, Violeta Raverdy, Julien Teigny, Hélène Verkindt, Maxence Debert, Philippe Preux, François Pattou, et al. Development and validation of an interpretable machine learning based calculator for predicting 5 year-weight trajectories after bariatric surgery: a multinational retrospective cohort SOPHIA study.

  The Lancet Digital Health, 2023

- Robert Caiazzo, Pierre Bauvin, Camille Marciniak, **PS**, et al. Impact of robotic assistance on complications in bariatric surgery at expert laparoscopic surgery centers. a retrospective comparative study with propensity score.

  Annals of Surgery, 2023

obesityweek | THE OBESITY SOCIETY
November 1-4, 2022 • San Diego

IFSO
NAPOLI
2023

Contributions in statistics and machine learning

Publications in peer-reviewed international conferences with proceedings

- Sayak Ray Chowdhury, **PS**, Odalric-Ambrym Maillard, and Aditya Gopalan. Bregman deviations of generic exponential families.
  In Conference On Learning Theory, 2023
- **PS** and Odalric-Ambrym Maillard. Risk-aware linear bandits with convex loss.
  In International Conference on Artificial Intelligence and Statistics, 2023
- Dorian Baudry, **PS**, and Odalric-Ambrym Maillard. From optimality to robustness: Adaptive re-sampling strategies in stochastic bandits.
  Advances in Neural Information Processing Systems, 2021b

Workshop presentations in peer-rewiewed international conferences

- **PS** and Odalric-Ambrym Maillard. Risk-aware linear bandits with convex loss.
  In European Workshop on Reinforcement Learning, 2022 (poster)

Ongoing

- **PS**. Empirical chernoff concentration: beyond bounded distributions
- **PS**, Romain Gautron, Odalric-Ambrym Maillard, Marc Corbeels, Chandra A. Madramootoe, and Nitin Joshi. Quantifying the uncertainty of crop management decisions based on crop model simulations

Software contributions

Bariatric Weight Trajectory Prediction

https://bwtp.univ-lille.fr

rlberry

https://github.com/rlberry-py/rlberry

concentration-lib

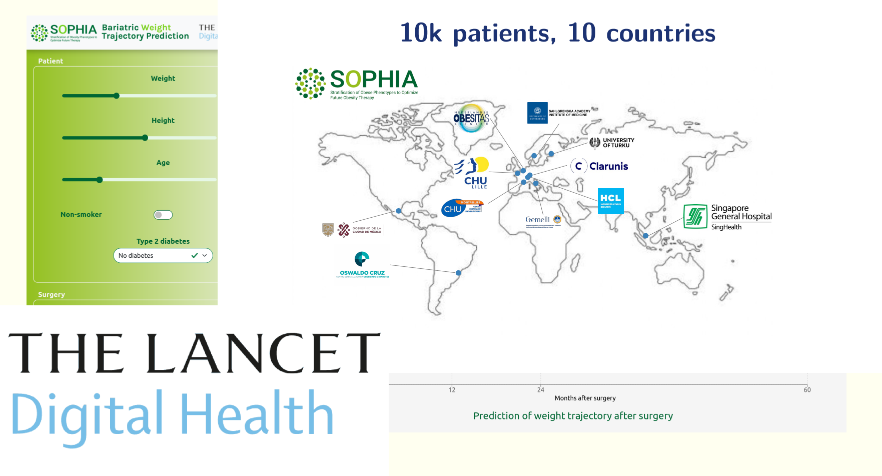
https://pypi.org/project/concentration-lib

Bariatric Weight Trajectory Prediction

Interpretable tree-based prediction model

PS, Bauvin et al., The Lancet Digital Health 2023.

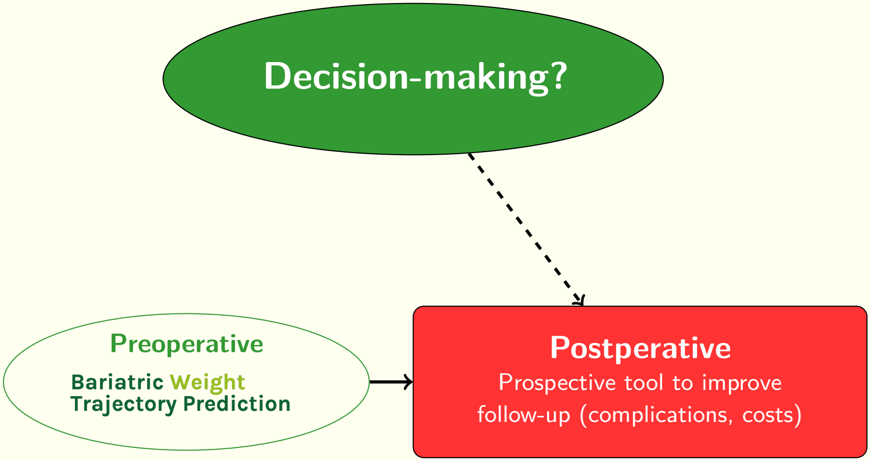
Bariatric Weight Trajectory Prediction



Interpretable tree-based prediction model

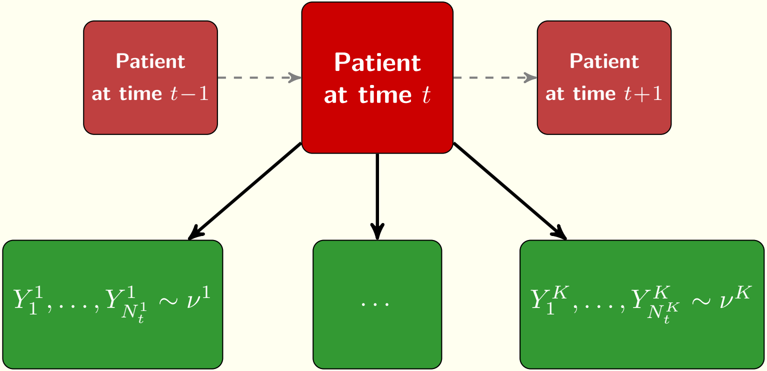
PS, Bauvin et al., The Lancet Digital Health 2023.

From preoperative to postoperative follow-up



An abstract framework for decision-making

Stochastic bandit



An abstract framework for decision-making

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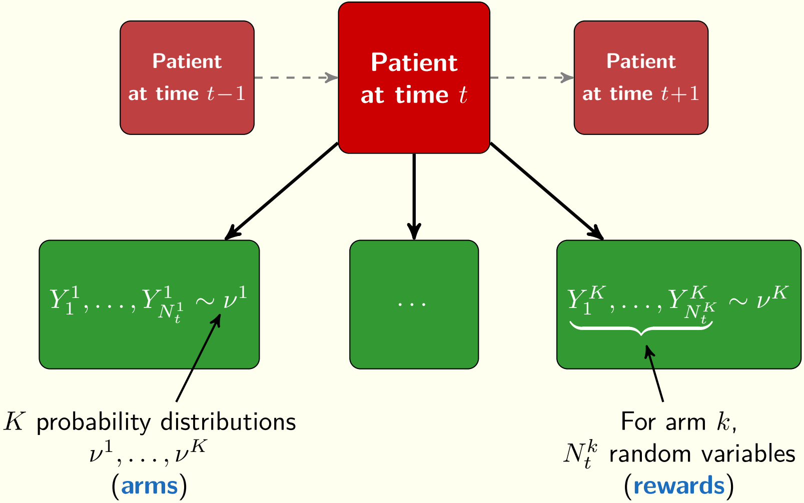

Stochastic bandits

$Y_1^1, \ldots, Y_{N_t^1}^1 \sim \nu^1$ | $\ldots$ | $\underbrace{Y_1^K, \ldots, Y_{N_t^K}^K} \sim \nu^K$

K probability distributions $\nu^1, \ldots, \nu^K$ (**arms**)

For arm k, N_t^k random variables (**rewards**)

- Active sampling of arm π_t.

Score (e.g. expectation)

- Minimise **regret** $\mathcal{R}_T = \sum_{t=1}^{T} \max_{k \in [K]} \rho(\nu^k) - \rho(\nu^{\pi_t})$.

- **Contextual bandits**: features X help predict rewards Y.

Lattimore and Szepesvári. Bandit algorithms. 2020.

Stochastic bandits

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K probability distributions
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👉 Regret lower bound ($T \to +\infty$):

$$\mathcal{R}_T = \Omega(\sqrt{T}) \quad \text{or} \quad \mathcal{R}_T \geqslant \log(T) \sum_{k \in [K]} \frac{\rho^\star - \rho(\nu^k)}{\mathcal{K}_{\inf}(\nu; \rho^\star)} \,.$$

Worst case

Instance-dependent

$$\mathcal{K}_{\inf}(\nu; \rho^\star) = \inf_{\nu'} \{\mathrm{KL}(\nu \parallel \nu'),\ \rho(\nu') > \rho^\star\}$$

Lattimore and Szepesvári. Bandit algorithms. 2020.

Exploration versus exploitation

$Y_1^1, \ldots, Y_{N_t^1}^1 \sim \nu^1$

$\ldots$

$\underbrace{Y_1^K, \ldots, Y_{N_t^K}^K} \sim \nu^K$

K probability distributions
$\nu^1, \ldots, \nu^K$
(**arms**)

For arm k,
N_t^k random variables
(**rewards**)

Greed is **not** good!

✖ $\pi_{t+1} \neq \underset{k \in [K]}{\operatorname{argmax}}\, \widehat{\rho}_t^k$.

Need to **explore**:

✔ Deterministic:
$\widehat{\rho}_t^k$ + confidence set.

✔ Randomised:
$\widehat{\rho}_t^k$ + noise.

Challenges of healthcare recommendations

Online advertising

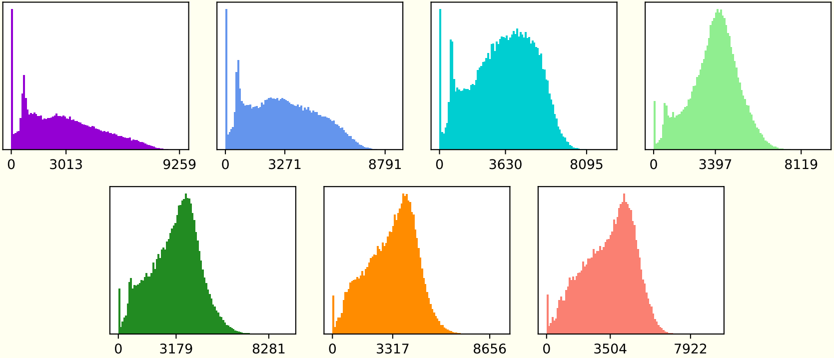
- ✔ Huge volume of data.
- ✔ Easy to model (Bernoulli, logistic...).
- ✔ Limited risks ($\rho = \mathbb{E}$).

Healthcare

- ✖ Slow and scarce data.
- ✖ Nonparametric models.
- ✖ Risky.

Example distributions

Grain yield distributions (=rewards)



I. Efficient deterministic exploration

- Quantify uncertainty for the score ρ from i.i.d. $Y_1, \ldots, Y_N$.

- ? Confidence sets: $\forall n \in \mathbb{N},\ \mathbb{P}\left(\rho \in \widehat{\Theta}_n\right) \geqslant 1 - \delta$ (e.g. $\delta = 5\%$).

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✔ **Anytime valid** confidence sequence:

$$\forall \text{ stopping time } N,\ \mathbb{P}\left(\rho \in \widehat{\Theta}_N\right) \geqslant 1 - \delta\,.$$

Ramdas, Grünwald, et al. Game-theoretic statistics and safe anytime-valid inference. Statistical Science, 2023.

I. Concentration bounds

Method	Assumption	Variance adaptive	Random N
t-test	Gaussian (or $N \to +\infty$)	✔	✖
Method of mixtures (MM) supermartingale inequality	Sub-Gaussian	✖	✔

Robbins and Pitman. Application of the method of mixtures to quadratic forms in normal variates. 1949.
Peña, Lai, and Shao. Self-normalized processes: Limit theory and statistical applications. 2009.

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Method of mixtures (MM) supermartingale inequality	Sub-Gaussian	✖	✔
Bregman MM	Exp. families	✔	✔
Empirical Chernoff MM	Second order sub-Gaussian	✔	✔

Robbins and Pitman. Application of the method of mixtures to quadratic forms in normal variates. 1949.
Peña, Lai, and Shao. Self-normalized processes: Limit theory and statistical applications. 2009.

I. Bregman concentration

Parameter

Exponential family: $p_\theta(y) = h(y)\exp\left(\langle \theta, F(y)\rangle - \mathcal{L}(\theta)\right)$.

Feature function

Log-partition function

Bregman divergence:

$$\mathcal{B}(\theta', \theta) = \mathcal{L}(\theta') - \mathcal{L}(\theta) - \langle \theta' - \theta, \nabla\mathcal{L}(\theta)\rangle = \mathrm{KL}(p_\theta \parallel p_{\theta'})\,.$$

Family	Known anytime valid concentration
Bernoulli, Gaussian (known variance)	✔
Gaussian, Chi-square, Poisson, Pareto, etc.	✖

Kaufmann and Koolen. Mixture martingales revisited. JMLR, 2021.

I. Bregman concentration

$$\widehat{\Theta}_n^\delta = \left\{\theta_0 \in \Theta : (n+c)\mathcal{B}_{\mathcal{L}}\left(\theta_0, \widehat{\theta}_{n,c}(\theta_0)\right) \leqslant \log\frac{1}{\delta} + \gamma_{n,c}(\theta_0)\right\}$$

is an anytime valid confidence sequence for this exponential family.

Chowdhury, **PS** et al., COLT 2023.

I. Bregman concentration

Regularised estimator

$$\widehat{\theta}_{n,c}(\theta_0) = (\nabla\mathcal{L})^{-1}\left(\frac{1}{n+c}\left(\sum_{j=1}^{n} F(Y_j) + c\nabla\mathcal{L}(\theta_0)\right)\right)$$

$$\widehat{\Theta}_n^{\delta} = \left\{\theta_0 \in \Theta : (n+c)\mathcal{B}_{\mathcal{L}}\left(\theta_0, \widehat{\theta}_{n,c}(\theta_0)\right) \leqslant \log\tfrac{1}{\delta} + \gamma_{n,c}(\theta_0)\right\}$$

Bregman divergence

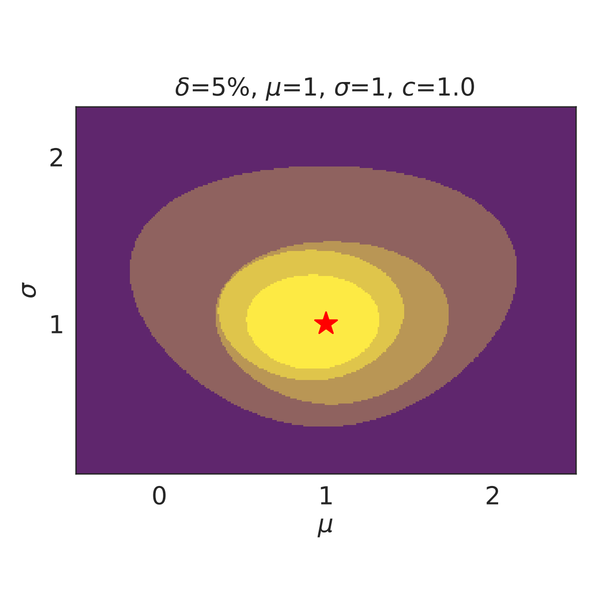
Bregman information gain

$$\gamma_{n,c}(\theta_0) = \log\left(\frac{\int_{\Theta}\exp\left(-c\mathcal{B}_{\mathcal{L}}(\theta', \theta_0)\right)d\theta'}{\int_{\Theta}\exp\left(-(n+c)\mathcal{B}_{\mathcal{L}}(\theta', \widehat{\theta}_{n,c}(\theta_0))\right)d\theta'}\right)$$

is an anytime valid confidence sequence for this exponential family.

Chowdhury, **PS** et al., COLT 2023.

I. Bregman concentration



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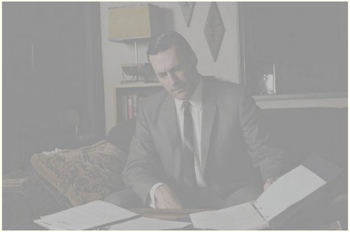
Challenges of healthcare recommendations



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- ✘ Nonparametric models.
- ✘ Risky.

II. Nonparametric: bootstrapping from data only

- Greedy policy:

Empirical average

$$\pi_{t+1} = \operatorname*{argmax}_{k \in [K]} \frac{1}{N_t^k} \sum_{j=1}^{N_t^k} Y_j^k \, .$$

II. Nonparametric: bootstrapping from data only

Empirical average

- Greedy policy:

$$\pi_{t+1} = \operatorname*{argmax}_{k \in [K]} \sum_{j=1}^{N_t^k} w_{j,t}^k Y_j^k \, .$$

$w_{j,t}^k = \frac{1}{N_t^k}$

II. Nonparametric: bootstrapping from data only

- Greedy policy with fictitious rewards:

Empirical average

$$\pi_{t+1} = \operatorname*{argmax}_{k \in [K]} \sum_{j=1}^{N_t^k} w_{j,t}^k Y_j^k + \widetilde{w}_t^k \widetilde{Y}_t^k \,.$$

$w_{j,t}^k, \widetilde{w}_t^k = \frac{1}{N_t^k + 1}$

Fictitious reward

Kveton, Szepesvari, et al. ICML, 2019.

II. Nonparametric: bootstrapping from data only

Randomised average

- Randomised greedy policy with fictitious rewards:

$$\pi_{t+1} = \operatorname*{argmax}_{k \in [K]} \sum_{j=1}^{N_t^k} w_{j,t}^k Y_j^k + \widetilde{w}_t^k \widetilde{Y}_t^k \,.$$

$w_{j,t}^k, \widetilde{w}_t^k \in [0,1]$ random

$$\sum_{j=1}^{N_t^k} w_{j,t}^k + \widetilde{w}_t^k = 1$$

Fictitious reward

Kveton, Szepesvari, et al. ICML, 2019.

II. Nonparametric: bootstrapping from data only

Nonparametric Thompson sampling:

Dirichlet average

$$\pi_{t+1} = \underset{k \in [K]}{\operatorname{argmax}} \sum_{j=1}^{N_t^k} w_{j,t}^k Y_j^k + \widetilde{w}_t^k \widetilde{Y}_t^k \,.$$

$w_{j,t}^k, \widetilde{w}_t^k \sim \mathrm{Dir}(1, \ldots, 1)$
(uniform distribution on the simplex)

Fictitious reward
($\approx$ reward upper bound)

Kveton, Szepesvari, et al. ICML, 2019.
Riou and Honda. ALT, 2020.

II. Nonparametric: bootstrapping from data only

☞ Nonparametric Thompson sampling:

Dirichlet average

$$\pi_{t+1} = \operatorname*{argmax}_{k \in [K]} \underbrace{\sum_{j=1}^{N_t^k} w_{j,t}^k Y_j^k}_{\text{Dirichlet average}} + \widetilde{w}_t^k \widetilde{Y}_t^k .$$

$w_{j,t}^k, \widetilde{w}_t^k \sim \mathrm{Dir}(1, \ldots, 1)$
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Fictitious reward
(≈ reward upper bound)

Beyond bounded rewards (with known bounds)?

Kveton, Szepesvari, et al. ICML, 2019.
Riou and Honda. ALT, 2020.

II. Regret guarantees for Dirichlet sampling

$$\pi_{t+1} = \underset{k \in [K]}{\operatorname{argmax}} \sum_{j=1}^{N_t^k} w_{j,t}^k Y_j^k + \widetilde{w}_t^k \widetilde{Y}_t^k \,.$$

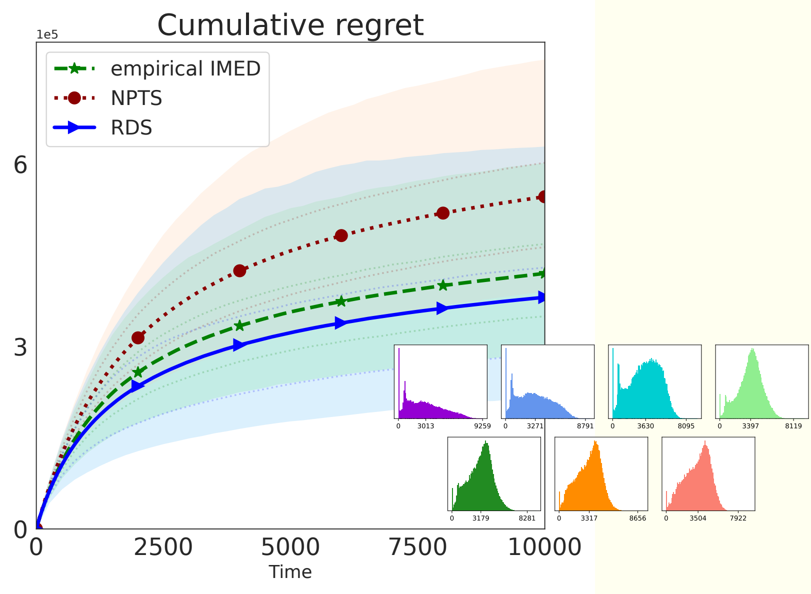
Regret guarantees (instance-dependent)

- Optimal in several settings
  (bounded, bounded detectable, semibounded + quantile condition)

Light tailed: $\mathcal{R}_T = \mathcal{O}\left(\log(T) \log\log(T)\right)$.

Baudry, **PS** et al., NeurIPS 2021.

II. Dirichlet sampling



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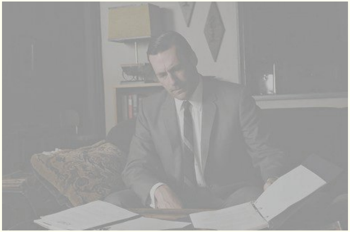
Challenges of healthcare recommendations



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III. From risk-neutral to risk-aware

☞ Non-contextual bandits:

- Maillard. Robust risk-averse stochastic multi-armed bandits.
  ALT, 2013.
- Baudry, Gautron, et al. Optimal Thompson sampling strategies for support-aware CVAR bandits.
  ICML, 2021a.

☞ Contextual bandits?

III. From risk-neutral to risk-aware

- Classical linear bandit $Y = \langle \theta^\star, X \rangle +$ zero-mean noise:

$$\widehat{\theta}_{t+1} = \min_{\theta \in \Theta} \sum_{s=1}^{t} \left(Y_s - \langle \theta, X_s \rangle\right)^2 \quad + \quad \text{exploration}\,.$$

Estimator of
$\rho(\nu) = \mathbb{E}_\nu \left[Y \mid X\right]$

Abbasi-Yadkori, Pál, and Szepesvári. Improved algorithms for linear stochastic bandits. NIPS, 2011.

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☞ Classical linear bandit $Y = \langle \theta^\star, X \rangle +$ zero-mean noise:

$$\widehat{\theta}_{t+1} = \min_{\theta \in \Theta} \sum_{s=1}^{t} \mathcal{L}\left(Y_s, \langle \theta, X_s \rangle\right) \quad + \quad \text{exploration}\,.$$

Estimator of
$\rho(\nu) = \min_{\theta} \mathbb{E}_\nu \left[\mathcal{L}\left(Y, \langle \theta, X \rangle\right) \mid X\right]$
(M-estimator)

$\mathcal{L}(y, \xi) = (y - \xi)^2$

Abbasi-Yadkori, Pál, and Szepesvári. Improved algorithms for linear stochastic bandits. NIPS, 2011.

III. From risk-neutral to risk-aware

- Elicitable risk measure linear bandit $Y \approx \langle \theta^\star, X \rangle +$ zero ρ-noise:

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$\mathcal{L}$ (strongly) convex mapping

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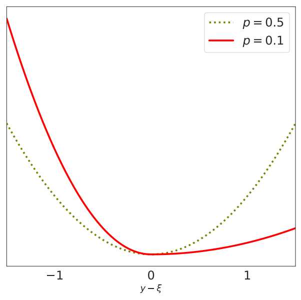
(M-estimator)

$\mathcal{L}$ (strongly) convex mapping

Expectile: $\mathcal{L}(y, \xi) = \left|p - \mathbb{1}_{y<\xi}\right| (y - \xi)^2$
($\neq$ loss for negative and positive rewards).



Abbasi-Yadkori, Pál, and Szepesvári. Improved algorithms for linear stochastic bandits. NIPS, 2011.

III. Risk-aware policies

- Deterministic exploration:

$$\pi_{t+1} = \mathrm{argmax}_x \left\langle \widehat{\theta}_{t+1}, x \right\rangle + \mathrm{UCB}(x),$$

Worst case: $\mathcal{R}_T = \widetilde{\mathcal{O}}\left(\sqrt{T}\right)$.

- Randomised Dirichlet exploration:

$$\widetilde{\theta}_{t+1} = \min_{\theta \in \Theta} \sum_{s=1}^{t} w_s^t \mathcal{L}\left(Y_s, \langle \theta, X_s \rangle\right) + \widetilde{w}_{t+1}^t \mathcal{L}\left(\widetilde{Y}, \langle \theta, \widetilde{X} \rangle\right),$$

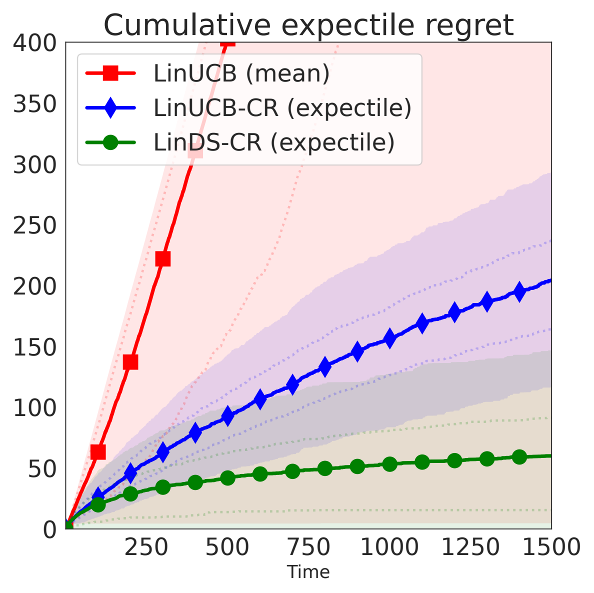
$$\pi_{t+1} = \underset{x}{\mathrm{argmax}} \left\langle \widetilde{\theta}_{t+1}, x \right\rangle,$$

Worst case: $\mathcal{R}_T \stackrel{?}{=} \widetilde{\mathcal{O}}\left(\sqrt{T}\right)$.

Linear Gaussian Thompson sampling?
(strong approximation of weighted bootstrap)

PS, Maillard, EWRL 2022; **PS**, Maillard, AISTATS 2023.

III. Risk-aware policies



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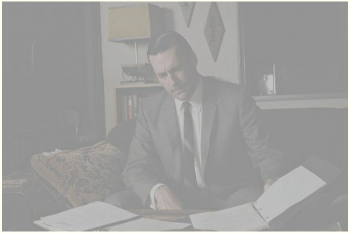
Challenges of healthcare recommendations



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Conclusion and future works

Preoperative

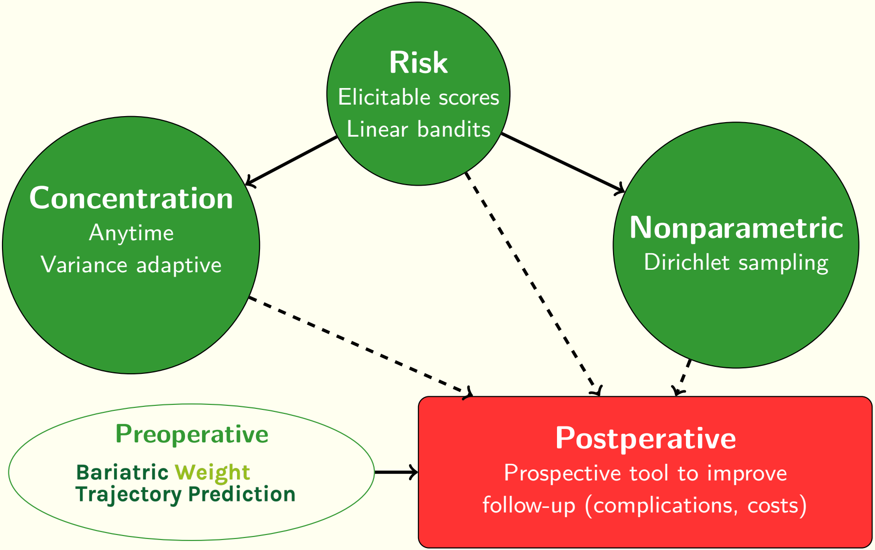
Bariatric Weight Trajectory Prediction

→

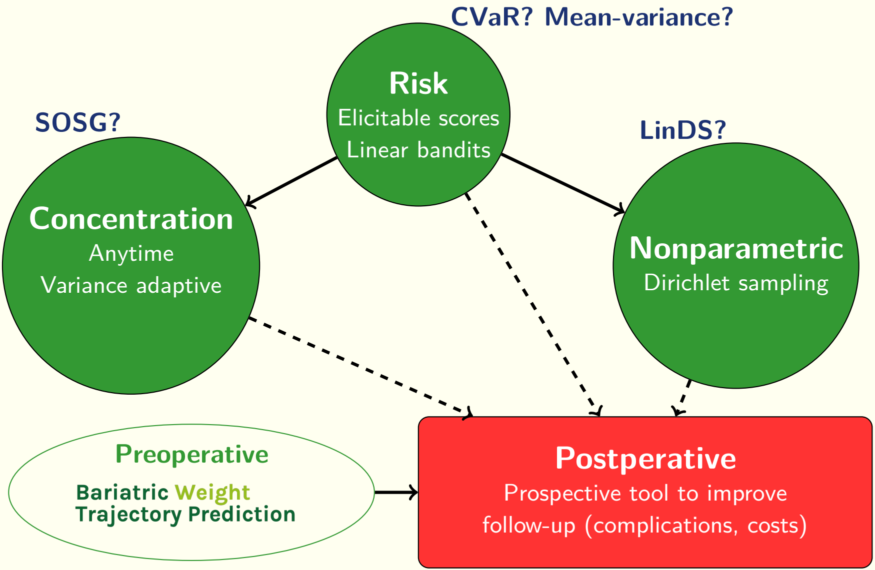
Postperative

Prospective tool to improve follow-up (complications, costs)

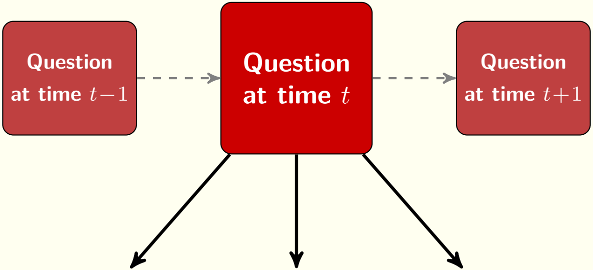
Conclusion and future works



Conclusion and future works



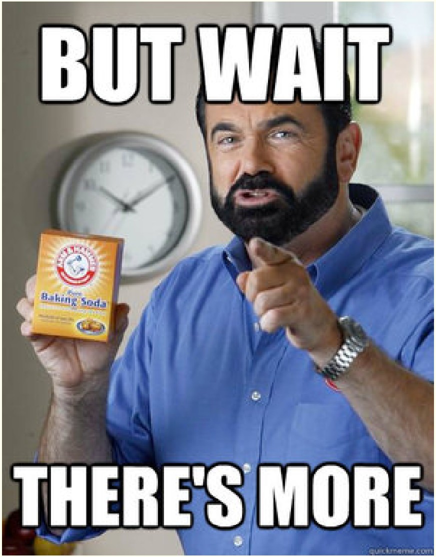
Questions



Angry jury

Happy jury

Confused jury


BUT WAIT
Baking Soda
THERE'S MORE
quickmeme.com

Empirical Chernoff concentration

- Sub-Gaussian (SG): $\forall \lambda \in \mathbb{R},\ \log \mathbb{E}\left[e^{\lambda(Y-\mu)}\right] \leqslant \frac{\lambda^2 R^2}{2}$.

- **Second order sub-Gaussian (SOSG):**
  $\forall \lambda \in \mathbb{R}_+,\ \log \mathbb{E}\left[e^{-\lambda(Y-\mu)^2}\right] \leqslant -\frac{1}{2}\log\left(1+2\lambda(\rho R)^2\right)$.

  Second to first order variance ratio ($\rho \in (0,1]$)

Family	Known anytime valid concentration
Gaussian (unknown variance)	✖
Uniform, symmetric triangular (unknown support)	✖
Other nonparametric distributions	✖

Empirical Chernoff concentration

$$\widehat{\Theta}_n^\delta = \left[\widehat{\mu}_n \pm \sqrt{\frac{2}{n}\left(1+\frac{\alpha}{n}\right) \underbrace{R^2}_{\text{Proxy variance}} \log\left(\frac{2\sqrt{1+\frac{n}{\alpha}}}{\delta}\right)}\right]$$

is an anytime valid confidence sequence for SG.

PS, TBA 2024.

Empirical Chernoff concentration

$$\widehat{\Theta}_{n,\rho}^{\delta} = \left[\widehat{\mu}_n \pm \sqrt{\frac{2}{n}\left(1+\frac{\alpha}{n}\right)\frac{\lfloor n/2 \rfloor V^{I}_{\lfloor n/2 \rfloor}}{\rho^2 \left(G^{\mathrm{T}}_{\beta,\gamma,\zeta,\lfloor n/2 \rfloor}\right)^{-1}\left(\frac{3}{\delta}\right)} \log\left(\frac{3\sqrt{1+\frac{n}{\alpha}}}{\delta}\right)}\right]$$

is an anytime valid confidence sequence for SOSG.

PS, TBA 2024.

Empirical Chernoff concentration

Empirical variance estimator

$$\widehat{\Theta}^{\delta}_{n,\rho} = \left[\widehat{\mu}_n \pm \sqrt{\frac{2}{n}\left(1+\frac{\alpha}{n}\right)\frac{\lfloor n/2\rfloor V^I_{\lfloor n/2\rfloor}}{\rho^2\left(G^{\mathrm{T}}_{\beta,\gamma,\zeta,\lfloor n/2\rfloor}\right)^{-1}\left(\frac{3}{\delta}\right)}\log\left(\frac{3\sqrt{1+\frac{n}{\alpha}}}{\delta}\right)}\right]$$

Second to first order variance ratio

$$G^{\mathrm{T}}_{\beta,\gamma,\zeta,\lfloor n/2\rfloor}(z) = \frac{U\left(\beta,\gamma+\frac{t}{2};\zeta+\frac{z}{2}\right)}{U(\beta,\gamma;\zeta)}$$

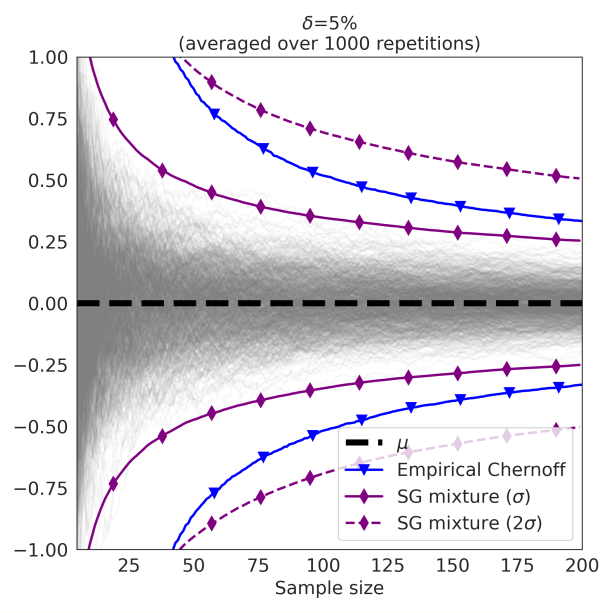
$$U(b,c;z) = \frac{1}{\Gamma(b)}\int_0^{+\infty} u^{b-1}(1+u)^{c-b-1}e^{-zu}\,du$$

(Tricomi's confluent hypergeometric function)

is an anytime valid confidence sequence for SOSG.

PS, TBA 2024.

Empirical Chernoff concentration



PS, TBA 2024.